Adaptive Analog Function Computation via Fading Multiple-Access Channels

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***Abstract*— In this letter, we propose an *adaptive* analog function computation (AFC) via *fading* multiple-access channels in which multiple sensors simultaneously send their observations and then the fusion center computes the desired function via the superposition property of wireless channels. In particular, each sensor adaptively sends its observation to the fusion center based on its *causal* channel state information (CSI). Numerical results show that the adaptive AFC significantly outperforms the conventional non-adaptive AFC in terms of the outage probability of function estimation error. The adaptive AFC operates in a fully distributed manner with local and causal CSI, applicable to various practical sensor network applications.**

***Index Terms*— Function computation, analog function computation, fading multiple-access channels, adaptive transmission, channel state information.**

I. Introduction

In various applications, the main purpose of communication in wireless sensor networks (WSNs) is to compute some pre-defined *functions* of sensor observations at the fusion center, rather than obtaining the observations themselves [1]. Substantial studies have been performed in the context of function computing for WSNs. In [2], linear source coding has been applied to efficiently compress each sensor observation for the function computation over Gaussian multiple-access channels (MACs). An efficient way of computing the modulo sum or sum of Gaussian sources over Gaussian MACs using lattice codes has also been proposed in [2] and [3]. This lattice-based computation has been extended to multiple receiver networks in which each relay computes or decodes linear combination of the sources, which is called compute-and-forward [4]. The linear source coding and lattice-based computation approach has been recently applied for computing the arithmetic sum and type functions in [5]–[7].

In [8] and [9], a simple computation strategy, called *analog function computation (AFC)*, has been investigated for wireless MACs. The AFC adopts both pre-processing at each sensor and post-processing at the fusion center for computing or estimating functions, in which all sensors concurrently participate in the transmission and the channel inversion technique is used at each sensor in order to compensate fading phenomenon [8]. The AFC was further analyzed in terms of the maximum achievable computation rate in clustered sensor networks [9]. In addition, AFC was extended to multiple-input multiple-output (MIMO) MACs with channel uncertainties [10].

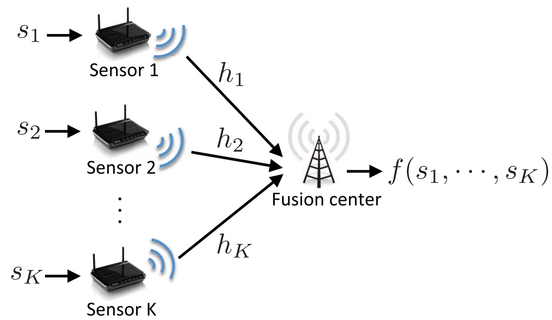

Fig. 1. Function computation via multiple-access channels.

In this letter, we propose a novel AFC technique that significantly improves the performance of the conventional AFC technique by efficiently exploiting the fading effect of wireless channels. For the proposed technique, each sensor adaptively sends its sensing information to the fusion center based on causal channel state information (CSI). Note that, for the AFC, each sensor's sensing information is conveyed by the received power at the fusion center. Hence each sensor should compensate its channel gain at the transmitter side when channel gains between sensors are unequal. For fading environment, however, the required transmit power for simple channel compensation applied in [11] can be arbitrarily large. For the best of our knowledge, this letter is the first result resolving such limitation of the AFC, essentially required for applying to fading environment.

Notations: The absolute value, Frobenius norm, transpose, and conjugate transpose operations are denoted by $|\cdot|$, $\|\cdot\|$, $(\cdot)^\dagger$, and $(\cdot)^\ddagger$, respectively. $\mathbf{1}_A$ denotes the indicator function of an event A, which becomes one if A happens or zero otherwise. $\text{diag}(a(1), \cdots, a(n))$ denotes the $n \times n$ diagonal matrix consisting of a_i as the ith diagonal element. In addition, $[1:n]$ denotes $\{1, 2, \cdots, n\}$.

II. Problem Formulation

A. System Model

We consider the problem of function computation via a *fading* MAC as depicted in Fig. 1. Each sensor $k \in [1:K]$ periodically observes its sensing data or measurement, denoted by $s_k \in [s_{\min}, s_{\max}]$, and the fusion center wishes to estimate the desired function of the K sensing data, denoted by $f(s_1, \cdots, s_K) \in [f_{\min}, f_{\max}]$, where $s_{\min} < s_{\max}$, $f_{\min} = \min_{(s_1,\cdots,s_K)\in[s_{\min},s_{\max}]^K}\{f(s_1,\cdots,s_K)\}$, and $f_{\max} = \max_{(s_1,\cdots,s_K)\in[s_{\min},s_{\max}]^K}\{f(s_1,\cdots,s_K)\}$.

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$$P_{k,t} = \begin{cases} P_{\mathsf{peak},k} & \text{if } |h_k(t)|^2 \geq \eta \text{ and } \sum_{i=1}^{t-1} |h_k(i)|^2 \mathbf{1}_{|h_k(i)|^2 \geq \eta} \leq \dfrac{\phi_k(s_k)}{P_{\mathsf{peak},k}} - |h_k(t)|^2, \\ \dfrac{\phi_k(s_k) - P_{\mathsf{peak},k} \sum_{i=1}^{t-1} |h_k(i)|^2 \mathbf{1}_{|h_k(i)|^2 \geq \eta}}{|h_k(t)|^2} & \text{if } |h_k(t)|^2 \geq \eta \text{ and } \dfrac{\phi_k(s_k)}{P_{\mathsf{peak},k}} - |h_k(t)|^2 < \sum_{i=1}^{t-1} |h_k(i)|^2 \mathbf{1}_{|h_k(i)|^2 \geq \eta} \leq \dfrac{\phi_k(s_k)}{P_{\mathsf{peak},k}}, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

For notational simplicity, the set of K sensing data is denoted by $\mathbf{s} = [s_1, \cdots, s_K]^\dagger$. Then, $f(\mathbf{s}) = f(s_1, \cdots, s_K)$. We assume that $\mathbf{s}$ is drawn from a continuous distribution. Let $\mathbf{S} = [S_1, \cdots, S_K]^\dagger$ denote the set of K random variables associated with $\mathbf{s}$ and $p_{\mathbf{S}}(\mathbf{s})$ denote the underlying joint probability density function of the K sensing data.[1] Notice that the desired function $f(\mathbf{S})$ is also a random variable induced by $\mathbf{S}$.

Suppose that the sensing period of each sensor is given by T time slots and, thus, the fusion center is required to estimate the desired function of K sensing data samples for every T time slots. For each sensing period, the received signal of the fusion center at time t is given by

$$y(t) = \sum_{k=1}^{K} h_k(t) x_k(t) + z(t) \quad (1)$$

for $t \in [1 : T]$, where $x_k(t)$ represents the transmit signal of sensor k at time t, $h_k(t)$ represents the wireless channel from sensor k to the fusion center at time t, and $z(t)$ denotes the additive noise of the fusion center at time t, which follows $\mathcal{CN}(0, 1)$ and is *independent* for different time slots. From (1), the received signal vector over T time slots at the fusion center can also be written as

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{z}, \quad (2)$$

where $\mathbf{y} = [y(1), \cdots, y(T)]^\dagger$, $\mathbf{x}_k = [x_k(1), \cdots, x_k(T)]^\dagger$, $\mathbf{H}_k = \text{diag}(h_k(1), \cdots, h_k(T))$, and $\mathbf{z} = [z(1), \cdots, z(T)]^\dagger$.

We assume identically and independently distributed (i.i.d.) Rayleigh fading, i.e., $h_k(t)$ follows $\mathcal{CN}(0, 1)$ and is independent for different k and t.[2] As seen in (1) and (2), we consider the simplified channel model ignoring the path-loss between each sensor and the fusion center in order to focus on the adaptation of the AFC based on short-term fading. We further assume that each sensor *causally* knows its channel coefficient, which can be estimated at each sensor based on the beacon signal sent from the fusion center by exploiting channel reciprocity. Hence, $x_k(t)$ can be a function of $\{h_k(i)\}_{i=1}^t$ for $t \in [1 : T]$. Each sensor k should satisfy both the *peak power constraint* and *average power constraint* during its transmission, i.e., $|x_k(t)|^2 \leq P_{\mathsf{peak},k}$ for all $t \in [1 : T]$ and $\mathbb{E}[|x_k(t)|^2] \leq P_{\mathsf{ave},k}$. Notice that if $P_{\mathsf{peak},k} \leq P_{\mathsf{ave},k}$, then the average power constraint $P_{\mathsf{ave},k}$ becomes inactive.

[1] Although we assume continuous sources here, the main result of this letter is also applicable for discrete sources.

[2] In this letter, we focus on delay-tolerant applications such as environmental monitoring and data collection in warehouse, which typically collects data from sensors infrequently during a long period of time. In this case, sensing data can be sent over multiple coherence-time periods so that $h_k(t)$ becomes time-varying.

B. Performance Metric: Function Estimation Error

In this letter, we adopt the outage probability of function estimation error as a performance metric as in [8]. For convenience, denote $\mathbf{x}_k(s_k) \in \mathbb{C}^T$ as the transmit signal vector of sensor k when $S_k = s_k$ and $\mathbf{y}(\mathbf{s})$ as the received signal vector at the fusion center when $\mathbf{S} = \mathbf{s}$. Let $\hat{f}(\mathbf{s}) : \mathbb{C}^T \to \mathbb{R}$ be the estimated desired function upon receiving $\mathbf{y}(\mathbf{s})$ at the fusion center. For given $\mathbf{s}$, the function estimation error is defined as $E(\mathbf{s}) = \frac{\hat{f}(\mathbf{s}) - f(\mathbf{s})}{f_{\max} - f_{\min}}$.

Then, the outage probability of function estimation error is given by

$$\Pr(|E(\mathbf{S})| \geq \epsilon) = \int_{\mathbf{s} \in [s_{\min}, s_{\max}]^K} p_{\mathbf{S}}(\mathbf{s}) \Pr(|E(\mathbf{s})| \geq \epsilon | \mathbf{S} = \mathbf{s}) \, d\mathbf{s}. \quad (3)$$

III. Adaptive Analog Function Computation

In this section, we propose an *adaptive* analog function computation scheme in which each sensor performs adaptive transmission based on its causal CSI in a distributed manner. The proposed technique adopts the basic transmit and receive structure of the conventional analog function computation proposed in [8].

A. Encoding at Each Sensor

Unless otherwise specified, assume that the kth sensing data is given by $S_k = s_k$ for $k \in [1 : K]$ from now on. Let $\phi_k(s_k) : [s_{\min}, s_{\max}] \to [0, \phi_{\max}]$ be the target received power required to deliver to the fusion center from sensor k, which maps s_k to a real value between zero and $\phi_{\max}$, where $\phi_{\max} = \max_{k \in [1:K], s_k \in [s_{\min}, s_{\max}]} \{\phi_k(s_k)\}$. In order to send at the target received power $\phi_k(s_k)$ to the fusion center, sensor k sets its transmit power at time t to $P_{k,t}(\phi_k(s_k), \{h_k(i)\}_{i=1}^t) : ([0, \phi_{\max}], \mathbb{C}^t) \to [0, P_{\mathsf{peak},k}]$ subject to $\mathbb{E}[P_{k,t}(\phi_k(s_k), \{h_k(i)\}_{i=1}^t)] \leq P_{\mathsf{ave},k}$. Finally, sensor k transmits $\mathbf{x}_k = \mathbf{A}_k \mathbf{u}_k$, where $\mathbf{A}_k = \text{diag}(\sqrt{P_{k,1}}, \cdots, \sqrt{P_{k,T}})$ and $\mathbf{u}_k = [e^{\theta_{k,1}}, \cdots, e^{\theta_{k,T}}]^\dagger$. Here $\theta_{k,t}$ is drawn uniformly at random from $[0, 2\pi)$ and independent for different k and t. That is, the information about s_k is delivered via $\mathbf{A}_k$ by using the length-T random phase sequence $\mathbf{u}_k$. Fig. 2 (a) illustrates the encoding structure of the proposed adaptive analog function computation scheme.

Notice that the proposed encoding process allows each sensor to adjust its transmit power in a distributed manner using its causal CSI. If we set $\phi_{\max}$ larger, the proposed analog function computation will be more robust against distortion due to the noise at the fusion center. However, the probability that each sensor fails to deliver its target received power $\phi_k(s_k)$

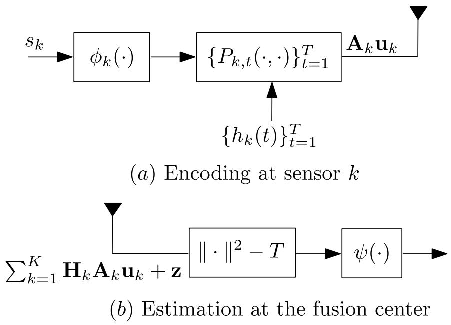

Fig. 2. Encoding and decoding structure of the proposed adaptive analog function computation.

to the fusion center will increase, which may cause another type of distortion (*power delivery failure*) at the fusion center because each sensor adapts according to only causal CSI and does not know the future CSI. Therefore, the technical challenge of the adaptive analog function computation is to determine $\phi_{\max}$ and the corresponding $\{\mathbf{A}_k\}_{k=1}^{K}$, while minimizing distortion due to both the noise and the power delivery failure. In Section III-B, we provide how to set $\phi_{\max}$ and $\{\mathbf{A}_k\}_{k=1}^{K}$. Furthermore, in Section III-D, we provide how to set $\{\phi_k(s_k)\}_{k=1}^{K}$ for a given $\phi_{\max}$.

B. Setting $\phi_{\max}$ and $\{\mathbf{A}_k\}_{k=1}^{K}$

For $t \in [1:T]$, each sensor k sequentially adjusts $P_{k,t}$ according to the causal CSI $\{h_k(i)\}_{i=1}^{t}$ as in (4) at the top of the previous page. In particular, sensor k transmits with a positive power at time t if its current channel gain is greater than a certain threshold $\eta \geq 0$, i.e., $|h_k(t)|^2 \geq \eta$, as seen from the first and second cases in (4). Notice that $P_{\mathsf{peak},k}\sum_{i=1}^{t-1}|h_k(i)|^2\mathbf{1}_{|h_k(i)|^2\geq\eta}$ is the received power from sensor k to the fusion center up to $t-1$ time slots when sensor k transmits symbols at the peak power. Hence, at time slot t, sensor k transmits with peak power, $P_{\mathsf{peak},k}$, if the current channel gain is larger than a certain threshold and the received power up to t time slots ($P_{\mathsf{peak},k}\sum_{i=1}^{t}|h_k(i)|^2\mathbf{1}_{|h_k(i)|^2\geq\eta}$) is still less than the target received power $\phi_k(s_k)$, which indicates the first case of (4). On the other hand, the second case of (4) represents the case that the received power up to t time slots to the fusion center becomes larger than $\phi_k(s_k)$ if sensor k transmits with power $P_{\mathsf{peak},k}$. In this case, sensor k transmits with the remaining power, $\frac{\phi_k(s_k)-P_{\mathsf{peak},k}\sum_{i=1}^{t-1}|h_k(i)|^2\mathbf{1}_{|h_k(i)|^2\geq\eta}}{|h_k(t)|^2}$, for satisfying the target received power from sensor k to the fusion center, $\phi_k(s_k)$. Obviously, the adaptive power allocation strategy in (4) satisfies the peak power constraint.

Now consider the average power constraint. Since the expected received power from sensor k to the fusion center when $|h_k(t)|^2 \geq \eta$ and $\sum_{i=1}^{t-1}|h_k(i)|^2\mathbf{1}_{|h_k(i)|^2\geq\eta} \leq \frac{\phi_k(s_k)}{P_{\mathsf{peak},k}} - |h_k(t)|^2$ (the first case in (4)) is given by $\mathbb{E}\left[|h_k|^2\big||h_k|^2\geq\eta\right]\cdot P_{\mathsf{peak},k} = (1+\eta)P_{\mathsf{peak},k}$ from the facts that

$$\Pr(|h_k(t)|^2\geq\eta) = \int_{\eta}^{\infty} e^{-x}dx = e^{-\eta},$$
$$\mathbb{E}\left[|h_k|^2\big||h_k|^2\geq\eta\right] = \int_{\eta}^{\infty} x\frac{e^{-x}}{\Pr(|h_k(t)|^2\geq\eta)}dx = 1+\eta, \tag{5}$$

the expected number of time slots of sensor k transmitting with a non-zero power for $t \in [1:T]$ is upper bounded by $\lceil\frac{\phi_k(s_k)}{(1+\eta)P_{\mathsf{peak},k}}\rceil$, where $\lceil\cdot\rceil$ denotes the ceiling operation. Hence, the expected total power consumption of sensor k is upper bounded by

$$P_{\mathsf{peak},k}\left(\frac{\phi_k(s_k)}{(1+\eta)P_{\mathsf{peak},k}}+1\right) = \frac{\phi_k(s_k)}{1+\eta}+P_{\mathsf{peak},k}. \tag{6}$$

Then, the average power constraint $P_{\mathsf{ave},k}$ can be satisfied for sensor k regardless of the realization of S_k if $\frac{\phi_{\max}}{1+\eta}+P_{\mathsf{peak},k} \leq TP_{\mathsf{ave},k}$. Therefore, we set

$$\phi_{\max} = (1+\eta)\min_{k\in[1:K]}\{TP_{\mathsf{ave},k}-P_{\mathsf{peak},k}\}. \tag{7}$$

Now consider the event that each sensor k fails to deliver its target received power $\phi_k(s_k)$ to the fusion center. Note that

$$\frac{\phi_k(s_k)}{\Pr(|h_k|^2\geq\eta)\mathbb{E}\left[|h_k|^2\big||h_k|^2\geq\eta\right]P_{\mathsf{peak},k}} = \frac{\phi_k(s_k)}{e^{-\eta}(1+\eta)P_{\mathsf{peak},k}} \tag{8}$$

is an approximate expected number of time slots that $\phi_k(s_k)$ is delivered to the fusion center when T is arbitrarily large, where the equality holds from (5). Hence, in order to avoid such event, (8) should be small enough compared to the communication period T for all $k \in [1:K]$. Therefore, we introduce a margin $\Delta \in [0,1]$ and set η to satisfy $\frac{\phi_{\max}}{e^{-\eta}(1+\eta)\min_{k\in[1:K]}\{P_{\mathsf{peak},k}\}} = T(1-\Delta)$, provided that

$$\begin{aligned}\eta &= \ln\left(\frac{(1+\eta)T(1-\Delta)\min_{k\in[1:K]}\{P_{\mathsf{peak},k}\}}{\phi_{\max}}\right)\\ &= \ln\left(\frac{T(1-\Delta)\min_{k\in[1:K]}\{P_{\mathsf{peak},k}\}}{\min_{k\in[1:K]}\{TP_{\mathsf{ave},k}-P_{\mathsf{peak},k}\}}\right),\end{aligned} \tag{9}$$

where the second equality holds from (7). In order to satisfy the condition $\eta \geq 0$, the feasible range of Δ is given by

$$\Delta \in \left[0, 1-\frac{\min_{k\in[1:K]}\{TP_{\mathsf{ave},k}-P_{\mathsf{peak},k}\}}{T\min_{k\in[1:K]}\{P_{\mathsf{peak},k}\}}\right]. \tag{10}$$

In summary, we set $\phi_{\max}$ as in (7) and η as in (9) within the range of Δ in (10).

C. Function Estimation at the Fusion Center

Recall that the information about the sensing data s_k is included in the target received power $\phi_k(s_k)$, which is added with others via wireless channels at the fusion center. Hence, the fusion center needs to measure the power of the received signal vector:

$$\begin{aligned}\|\mathbf{y}\|^2 &= \left(\sum_{k=1}^{K}\mathbf{H}_k\mathbf{A}_k\mathbf{u}_k+\mathbf{z}\right)^{\ddagger}\left(\sum_{l=1}^{K}\mathbf{H}_l\mathbf{A}_l\mathbf{u}_l+\mathbf{z}\right)\\ &= \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{t=1}^{T}\Big(\mathbf{1}_{|h_k(t)|^2\geq\eta}\mathbf{1}_{|h_l(t)|^2\geq\eta}\sqrt{P_{k,t}}\sqrt{P_{l,t}}\\ &\quad\cdot h_k(t)^{\ddagger}h_l(t)e^{\theta_{k,t}}{}^{\ddagger}e^{\theta_{l,t}}\Big)+\sum_{k=1}^{K}(\mathbf{H}_k\mathbf{A}_k\mathbf{u}_k)^{\ddagger}\mathbf{z}\\ &\quad+\sum_{l=1}^{K}\mathbf{z}^{\ddagger}\mathbf{H}_l\mathbf{A}_l\mathbf{u}_l+\mathbf{z}^{\ddagger}\mathbf{z}.\end{aligned} \tag{11}$$

Assuming that each sensor k delivers its target received power $\phi_k(s_k)$, which can be guaranteed with high probability by properly setting Δ in (9), then we have

$$\mathbb{E}[\|\mathbf{y}\|^2] = \sum_{k=1}^{K}\phi_k(s_k) + T \tag{12}$$

from the facts that

$$\begin{aligned}
&\mathbb{E}\Bigg[\sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{t=1}^{T}\Big(\mathbf{1}_{|h_k(t)|^2\geq\eta}\mathbf{1}_{|h_l(t)|^2\geq\eta}\sqrt{P_{k,t}}\sqrt{P_{l,t}} \\
&\qquad\cdot h_k(t)^{\ddagger}h_l(t)e^{\theta_{k,t}{}^{\ddagger}}e^{\theta_{l,t}}\Big)\Bigg] \\
&\overset{(*)}{=}\mathbb{E}\Bigg[\sum_{k=1}^{K}\sum_{t=1}^{T}\mathbf{1}_{|h_k(t)|^2\geq\eta}P_{k,t}|h_k(t)|^2\Bigg] = \sum_{k=1}^{K}\phi_k(s_k), \\
&\times\mathbb{E}\Bigg[\sum_{k=1}^{K}(\mathbf{H}_k\mathbf{A}_k\mathbf{u}_k)^{\ddagger}\mathbf{z}\Bigg] = \mathbb{E}\Bigg[\sum_{l=1}^{K}\mathbf{z}^{\ddagger}\mathbf{H}_l\mathbf{A}_l\mathbf{u}_l\Bigg] = 0, \\
&\times\mathbb{E}[\mathbf{z}^{\ddagger}\mathbf{z}] = T,
\end{aligned} \tag{13}$$

where $(*)$ holds from the facts that $\theta_{k,t}$ is independent for different k and also independent of channel coefficients and $\mathbb{E}[e^{\theta_{k,t}{}^{\ddagger}}] = \mathbb{E}[e^{\theta_{l,t}}] = 0$.

Note that $\|\mathbf{y}\|^2 - T$ becomes an *unbiased* estimator of $\sum_{k=1}^{K}\phi_k(s_k)$. Now consider the mapping function $\psi(\|\mathbf{y}\|^2 - T) : \mathbb{R} \to \mathbb{R}$ satisfying that $\psi(\sum_{k=1}^{K}\phi_k(s_k)) = f(\mathbf{s})$, which can be established for a broad class of fundamental functions including the arithmetic and geometric means, see [8, Sec. II] for more details. Then, the fusion center estimates the desired function as $\hat{f} = \psi(\|\mathbf{y}\|^2 - T)$. Fig. 2 (b) illustrates the function estimation procedure of the proposed adaptive analog function computation technique. Since the proposed adaptive AFC is implementable in a distributed manner, the coarse block-synchronization is enough. For more details, we refer to [8, Fig. 2] and the related explanation.

D. Examples: Arithmetic & Geometric Mean

In this subsection, we briefly state how to set $\{\phi_k(\cdot)\}_{k=1}^{K}$ and $\psi(\cdot)$ for computing the arithmetic and geometric means, which is also provided in [8, Sec. II].

1) Arithmetic mean: Suppose that $f(\mathbf{s}) = \sum_{i=1}^{K}s_k$. For this case, set $\phi_k(x) = \frac{\phi_{\max}(x - s_{\min})}{s_{\max} - s_{\min}}$ for $x \in [s_{\min}, s_{\max}]$ and $\psi(y) = \frac{s_{\max} - s_{\min}}{\phi_{\max}}y + Ks_{\min}$ for $y \in \mathbb{R}$. Obviously, $\psi(\sum_{k=1}^{K}\phi_k(s_k)) = f(\mathbf{s})$ is satisfied.

2) Geometric mean: For $s_{\min} > 0$, suppose that $f(\mathbf{s}) = (\prod_{i=1}^{K}s_k)^{1/K}$. For this case, set $\phi_k(x) = \frac{\phi_{\max}}{\ln(s_{\max})}\ln(x)$ for $x \in [s_{\min}, s_{\max}]$ and $\psi(y) = e^{\frac{\ln(s_{\max})}{\phi_{\max}}y}$ for $y \in \mathbb{R}$. Again, $\psi(\sum_{k=1}^{K}\phi_k(s_k)) = f(\mathbf{s})$ is satisfied.

IV. Simulation Results

We perform extensive computer simulations in order to demonstrate the improved *outage probability*, $\Pr(|E(\mathbf{S})| \geq \epsilon)$ in (3), of the proposed adaptive analog function computation technique over the conventional (non-adaptive) analog function computation technique in [8]. In particular, the conventional scheme corresponds to the case where $\eta = 0$ in the proposed scheme. We consider the arithmetic and geometric means as desired functions. We assume that each S_k is drawn i.i.d. from the uniform distribution in (0, 10] and set $P_{\mathsf{peak},k} = 10$ dB and $P_{\mathsf{ave},k} = 5$ dB for $k \in [1 : K]$ for simulations.

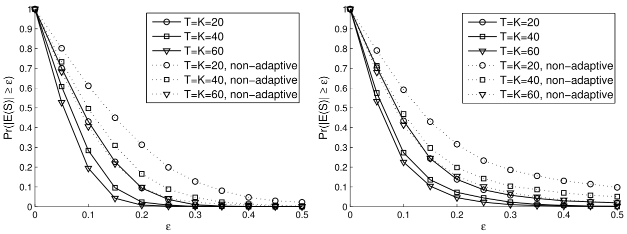

Fig. 3. Outage probability with respect to ϵ for the arithmetic mean (left) and for the geometric mean (right).

Fig. 3 shows the outage probability with respect to tolerable function computation error, ϵ. In simulation, the margin Δ of the proposed scheme is numerically optimized within the feasible range in (10) to minimize the outage probability. For comparison, we also plot the conventional non-adaptive scheme [8]. Note that the solid lines indicate the proposed technique and the dotted lines indicate the conventional technique. As seen in the figure, the proposed technique significantly outperforms the conventional technique in terms of the outage probability. When $T = K = 20$ and $\epsilon = 0.2$, for instance, the outage probability of the proposed technique is equal to 0.09, while the outage probability of the non-adaptive technique is equal to 0.31 for the arithmetic mean computation. It is also observed that the proposed technique significantly improves the outage probability for the geometric mean computation.

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